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HYDROGRAPHIC/ TOPOGRAPHIC CENTER WASHI.. J R ACKERET
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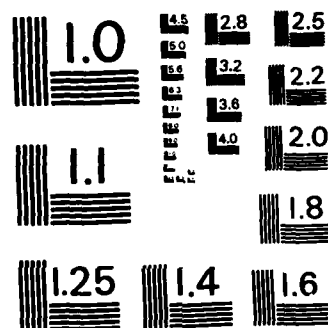
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AN INTERACTIVE FORTRAN PROGRAM FOR DETERMINING
RELIABILITY OF PSEUDO-RANGE GEODETIC POINT POSITIONING
USING THE GLOBAL POSITIONING SYSTEM

by

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Prepared for

Defense Mapping Agency

Washington, DC 20315

and

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Columbus, Ohio 43210

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DEDICATION

To Valerie and Claire

PREFACE

The development, implementation, test and evaluation of an interactive FORTRAN program for Covariance analysis was part of the GPS software development project at DMAHTC, Geodesy and Surveys Department, Techniques Office (GST).

It was performed under the supervision of Ben Roth, Chief, GST and Professor Dhaneshwar Hajela, Department of Geodetic Science, The Ohio State University.

This report was submitted to the Graduate School of The Ohio State University as partial fulfillment of the requirements for the Master of Science degree in Geodetic Science.

ABSTRACT

A transportable, generic, interactive FORTRAN program for the covariance analysis of pseudo-range experiments in the Global Positioning System (GPS) is presented for use in simulation studies, experiment planning and systems development. The interactive mode is especially suited for covariance analyses involving multiple parameters, by allowing ease of operation for data production and providing savings in time when computing geodetic point positioning.

The mathematical models (presented in Chapter 3) on which the program is based provide an overview of the GPS point positioning process, emphasising the areas of interest to geodesy. Also, special emphasis is placed upon station tracking geometry. The FORTRAN program allows for understanding correlation coefficients between the various parameters. A separate model suitable for covariance analysis is also presented.

This interactive FORTRAN program has been implemented on the UNIVAC at DMAHTC. The program was tested with simulated GPS data. The results indicate that the program is effective and efficient in helping to establish the reliability of the GPS point positioning technique. The program can be used in simulation studies, planning test and evaluation experiments, and GPS software development. The question/answer method has been used to develop this interactive program which enables the user to

sit at the computer terminal (scope) and rapidly assess the expected accuracy of the GPS point positioning technique as applied to a particular survey of interest .

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I thank also my science advisor, Mr. Brett Merritt, Chief of Satellite Geophysics Division Office for suggesting the need for this study and helping me think through the theory on which this study is based.

I wish to express my gratitude to all the professors at the University, especially Ivan Mueller and Richard Rapp for developing and clarifying the concepts presented in my academic program in geodetic science.

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LIST OF ACRONYMS AND ABBREVIATIONS

1. INTRODUCTION

"...Many users of satellite point positioning have adopted a wait-and-see attitude toward GPS..."

(Brown, 1979).

1.1 Background

The Defense Mapping Agency Hydrographic/Topographic Center (DMAHTC) provides mapping, charting and geodetic (MC&G) support to the Department of Defense (DoD) and other federal agencies.

The Geodesy and Surveys Department (GSD), DMAHTC, has the mission to acquire and reduce satellite tracking data to support MC&G requirements.

Currently, GSD utilizes the Navy Navigation Satellite System (NNSS) to support MC&G requirements. This utilization includes: operation and maintenance of a worldwide satellite tracking network (TRANET); data communication, processing and computation of the precise ephemerides and the deployment, operation and maintenance of portable satellite receivers (Geoceivers and MX 1502s) for the determination of precise geodetic positioning points and relative positions.

In the future, GSD will switch to the NAVSTAR Global Positioning System to support MC&G requirements when it replaces NNSS in the late 1990s as DoD's prime satellite navigation system.

1.2 System Description

A description of the system should begin with the NAVSTAR Global Positioning System (GPS). GPS is a space based radio navigation system under development by DoD, GPS Joint Program Office (JPO). The system is being developed to support the military forces' need to precisely compute their position, velocity and time any place on or near the earth, 24 hours of every day, under all weather conditions, and within a common reference frame.

GPS completed the concept validation (Phase I) in 1979 and is currently in full-scale development (Phase II). The system development schedule calls for an initial operational capability (IOC) in the 1986-87 time frame and a full operational capability (FOC) in the 1988-1989 time frame. At FOC, GPS will replace NNSS as DoD's prime satellite navigation system.

GPS consists of 3 parts, a Space Segment, a Control Segment and a User Segment. [Ward, 1981].

Space Segment

The operational Space Segment will consist of 18 satellites configured into 6 equally spaced 55 degree inclination

orbit planes. Each plane will contain 3 equally spaced satellites which are each in 12 hour, nearly circular (26,000 kilometer radius) orbits. The resulting satellite constellation will provide on or near the earth, continuous user visibility to a minimum of 4 satellites. In addition to the 18 satellites, a number (to be determined) of spare satellites will be in orbit to support continued FOC in the event of failures. The launch of the operational satellites will be accomplished with the Space Shuttle commencing in 1986.

The Space Segment of Phase I and Phase II comprises 4-5 developmental satellites which are in 12 hour, nearly circular (26,000 kilometer radius), 63 degree inclination orbits. The satellites are configured to provide 4 satellite visibility at Yuma Proving Ground, Arizona (YPG), which is the principal test location for the User Segment.

The GPS satellite contains a S band receiver, a L band transmitter, a solar/battery power system, a signal processor/generator and a redundant system of precise frequency/time standards. The satellite continuously broadcasts spread spectrum signals at center line frequencies of 1575.42 MHz (L1) and 1227.60 MHz (L2). These signals are encoded with the satellite's ephemerides and clock estimates, which are transmitted daily to the satellite from the Control Segment.

Control Segment

The Operational Control Segment will comprise five monitor stations (MSs), three upload stations and an operational control station (OCS). The MSs will be located worldwide and

will track the GPS satellites obtaining measurements of range and integrated Doppler at L1 and L2. These data will be communicated to the OCS where they will be processed to provide precise predictions of the satellites' ephemerides and clock estimates. These predictions will then be communicated to the upload stations where they will be transmitted to the satellites via S-band for broadcast to the User Segment.

The Control Segment of Phase I and Phase II is comprised of four MSs, one upload station and a Master Control Station (MCS). The upload station, MCS and an MS are collocated at Vandenberg AFB, California. The remaining three MSs are located in Alaska, Hawaii and Guam. This configuration supports User Segment testing at YPG.

The development and implementation of the Operational Control Segment will be phased. The operational MSs will be deployed in late 1984 and interfaced with an Intermediate Control Station (ICS) which will be collocated with the developmental MCS at Vandenberg AFB. In 1986, the OCS will begin to be phased in and will be located at a site in Colorado along with an upload and monitor station. In 1987, the OCS will take over the system and ICS will remain as a backup system.

User Segment

The operational User Segment consists of a family of user equipment (UE) which will support a variety of service-peculiar navigation requirements. The user equipment will comprise antennae, receivers, navigation processors and flexible modular interfaces (FMIs). The FMIs support the integration of the UE

with aircraft, ships and ground vehicles. These UE are designed to provide the user with real-time 15 meter navigation fix independent of the vehicles' dynamics, when GPS reaches FOC.

1.3 Purpose of this Study

The major objective of this study is to present a variance-covariance analysis for use in systems development, simulating and planning geodetic point positioning using NAVSTAR Global Positioning System (GPS). An interactive program is offered with an explanation of the theory and mathematical models on which the program is based. An overview of GPS for those interested in applying the pseudo-range method for geodetic activities is also presented.

The variance-covariance study is based on the method of estimate-consider analysis of parameters. The "best" estimation of a vector of unknown parameters, X , usually computed via a formulation of the state of the system, called estimate parameters, these being the latitude, longitude and height of a station in a common reference frame. Thus, one must "consider" the effect of the unestimated parameters, called consider parameters, which are tropospheric correction, the satellite's long-track, cross-track and radial track, and the bias, drift, and aging of the combined user-satellite clock parameters. The program is written in the interactive mode to allow the user to input the necessary estimate and consider parameters for analysis of the standard deviations of each plus the correlation factor between each parameter. The consider covariance shows the effect of the consider parameter on the estimate parameter

[Bierman, 1977]. Plots are supplied with the simulation study indicating optimal satellite tracking time for the establishment of the geodetic point using GPS point-positioning. Also, the program allows the user to plot the standard deviation of estimate parameters and correlation coefficients between each estimate parameter.

The choice of the parameter set was influenced by previous studies by Milliken and Zoller (1980) and Fell (1980). The primary emphasis is placed on the effect of consider parameters on the estimate parameters of a geodetic point. (see list of acronyms)

The introduction of an interactive program for the variance-covariance study allows the user for the first time to simulate a geodetic positioning experiment and view the results in real time. It was decided to write the program in a generic sense for future use since the mathematical models can easily be expanded or changed.

This study also illustrates a technique which allows the geodesist to analyze station selection for optimal network development. It provides the geodesist with the tools for understanding the process of measurement and the capability to partake in future simulations of experiments and planning for geodetic point positioning. With this information the geodesist can address the problem areas and implement sound statistical analysis of data and development of improved mathematical models.

1.4 Organization and Scope of Study

Chapter 2 covers the basic geometry of ground station tracking and a study of Geometric Dilution of Precision (GDOP). The description of this geometric study involves the mathematics for calculation of visibility circles, GPS sub-satellite points, Position Dilution of Precision (PDOP) and optimal ground station tracking geometry for any station in the world. The value of GDOP itself is a composite measure that reflects the influence of satellite geometry on the combined accuracy of the estimate of user time and user position. The value of PDOP is that value that reflects the influence of satellite geometry precluding the estimate of user time. Graphs are supplied showing the lowest GDOP value for tracking GPS satellites from a ground station over a 24 hour period indicating optimal ground station tracking for a GEOSTAR GPS user. The region around a ground station where a GPS satellite is visible, determined by station coordinates and the semi-major axis of satellite, is a visibility circle. An explanation of the interactive program for future use in studies involving the optimization of ground-station tracking of GPS satellites is given.

In Chapter 3 the mathematical models used in the variance-covariance study are described as well as future possible model refinements. A model, suitable for covariance analyses, is presented for determining the correlation coefficients between the estimate and consider parameters. A sensitivity study using estimate and consider parameters from which geodetic parameters

are estimated is also described. A summary of the sensitivity study is discussed and illustrated by using graphics developed on the Hewlett Packard 9845 Desktop Computer.

It was found that the models discussed in Chapter 3 do provide solutions to geodetic positioning using GPS pseudo-range measurements. These results are discussed in Chapter 4 along with recommendations for further investigations. The last chapter, chapter 5, also addresses some of the problems involved in the development and use of the models which will be created by users of the program.

The paper contains the following graphs: PDOP and GDOP, sub-satellite points on a world map, visibility circles on a world map, elevation angle vs time, azimuth vs elevation angle, and standard deviations of parameters and correlation coefficients among parameters vs time. Appendix A contains a listing of variance-covariance interactive FORTRAN program. Appendix B contains a sample run as viewed at the interactive computer terminal screen.

2. GEOMETRIC SIMULATION STUDY

2.1 Introduction

One of the main reasons for the development of NAVSTAR/Global Positioning System (GPS) is to provide GPS users with a precise navigational fix at any point anywhere in the world. This chapter describes a rapid method for performing the geometric simulation of station tracking for any point on earth. In the future when the NAVSTAR/GPS is fully operational, the simulation method described in this section will not only provide a rapid navigational fix to any observer at any point in the world, but also allow the user to verify that the geodetic receiver has selected the most appropriate satellites to ensure proper data acquisition.

The procedure for simulating a GPS satellite ephemeris is discussed first. The graphic section in the appendix illustrates world-wide NAVSTAR/GPS sub-satellite ground track points, and the visibility circles of the possible tracking stations the Defense Mapping Agency (DMA) will deploy for the testing of the GEOSTAR geodetic positioning receiver. Secondly, the graphics section shows strength of the geometry for the NAVSTAR/GPS constellations for these respective tracking stations. In doing this analysis, we describe the principle of Geometric Dilution of Precision (GDOP) and demonstrate the results using computer graphics. Finally, there is a brief discussion of the GPS receiver, GEOSTAR, that will be utilized by DMA in the near future for geodetic point positioning.

2.2 GPS Keplerian Orbit Generation

We describe the orbit generation program in this section. The first step in the procedure is the location of sub-satellite points, and the production of visibility circles. The Keplerian elements of each satellite from NSWC and the beginning and the ending epoch for the orbit generation are read into a subroutine that converts the NSWC orbital elements into the traditional Keplerian orbit elements. The value for the rate of change for a , e , and i are negligibly small. Then, the necessary input such as GM , ae , inverse flattening, and angular velocity of the earth's rotation, ω_e , (WGS 72) plus the six Keplerian elements are also read into the front part of the program.

The epoch for the orbit generation observations was 1983, Day 17, 0^h , 0^m , 0.0 sec. The earth centered, earth-fixed position and velocity vectors are computed by the program for each GPS satellite at 5 minute intervals. [Mueller, 1982]. The program is interactive so that the user can easily change the time span and number of intervals/unit of time of the orbit generation. The program has been tested by a comparison with NSWC ephemerides, and found to be within millimeter precision for several satellites at the epoch 1983, Day 17, 0^h 0^m 0.0 sec.

The program also has the capability for updating Keplerian elements so that the user could reasonably generate ephemerides without current NSWC elements. This method is outlined here and it may be used if a simulation study needs to be done for point-positioning tracking projects in the future.

First, one must update the argument of perigee, ω , right ascension of ascending node, Ω , and the mean anomaly. The value for the rate of change for a , e , and i are negligibly small. The rate of change for these elements, due to the second zonal coefficient C_{20} in the earth's gravitational potential, is given by [KAULA 1966, page 39]

$$\frac{dM}{dt} = \bar{n} - \frac{3 \pi C_{20} a_e^2}{4(1-e^2)^{3/2} a^2} (3\cos^2 i - 1) \quad (2.2 -1)$$

$$\frac{d\Omega}{dt} = \frac{3 \pi C_{20} a_e}{2(1-e^2)^2 a^2} \cos(i) \quad (2.2 -2)$$

$$\frac{d\omega}{dt} = \frac{3 \pi C_{20} a_e}{4(1-e^2)^2 a^2} (1-5\cos^2 i) \quad (2.2 -3)$$

where a_e is earth's mean equatorial radius, a , e , i , n are respectively the semi-major axis, eccentricity, inclination, and mean motion of the satellite orbit.

During the program run, the program asks if the elements need to be updated to a certain day; and then performs the update in a subroutine by multiplying the day change by the rate of change for each Keplerian element. One interesting finding was the lack of contribution given by the mean anomaly to the ephemeris caused by the perturbations of the satellite. The way this came

about involved the need for a requested station tracking geometry and study to verify the rise time of satellites with another agencies updated ephemeris. This simulation created the first opportunity to do the geometry analysis with cross checking the data utilizing ω, Ω and ω, Ω, M orbit parameter update. Interestingly, the geometry calculated using ω, Ω , parameter update was more correct and obviously quicker to compute than with all three.

2.2.1 Sub-Satellite Points

The procedure for calculation of sub-satellite points is performed by using a method developed by B. R. Bowring. [1976 Survey Review No. 181]

The program procedure for calculating the sub-satellite points is outlined as follows:

1) Input X,Y,Z of satellite; also inverse flattening, semi-major, a, and semi-minor, b, axes of reference ellipsoid and a test condition for iteration of ellipsoidal latitude, .

2) Solve for ZQ and XQ where

$$ZQ = ZA$$

$$XQ = (XA + YA)1/2$$

3) Solve for reduced latitude,

$$\beta = \text{DATAN2}(b*ZQ, a*XQ)$$

4) Set iteration to zero and intermediate value, ("store"),
to zero

5) Compute tangent of PHI

$$\tan \phi = (ZQ + e'^2 * b * (d \sin \beta)^3) / (XQ - a * e'^2 * (d \cos \beta)^3)$$

6) Test condition of PHI for iterative process.

if [DABS (PHI - STORE).LT.TESTC. OR .ITER.GT.10] Go to

Step 9

If not continue with Step 7.

7) Put store equal to PHI

STORE = PHI

8) Compute BETA.

$$\beta = \text{DATAN2}[(1 - e'^2)^{1/2} * \tan \phi]$$

Go To Step 4

9) Compute λ , Geodetic Longitude, $\lambda = \text{TAN}(YA, XA)$

This program produces a file for each satellite's ground track points so that the points can be plotted on a world map using a Hewlett Packard 9845 desktop computer. The plots are found in the following sections. In this manner, one can obtain graphics illustrating the optimal time for observing the present GPS constellations for any ground station. (See sample plots in this chapter). These graphs show the relative positions of each satellite in the current five satellite configuration. The azimuth versus elevation angle versus time plots add confidence to my PDOP and GDOP measurements because one can see that as the satellites spread out over the station, the value of GDOP and PDOP decreases.

2.2.2 Visibility Circles

In this section, the geometry and mathematics of GPS visibility circles for specific ground stations is presented. Given the geocentric coordinates, and R_e the mean radius of the earth, the coordinates convert in the following way:

$$(1) \quad Z_{STN} = R_e * \sin \phi$$

$$(2) \quad REQ = R_e * \cos \phi$$

(3) Determine values for X_{STN} , and Y_{STN} , Thus we have,

(See Figure 2.2-1)

$$X_{STN} = REQ * \cos \lambda$$

$$Y_{STN} = REQ * \sin \lambda$$

$$Z_{STN} = R_e * \sin \phi$$

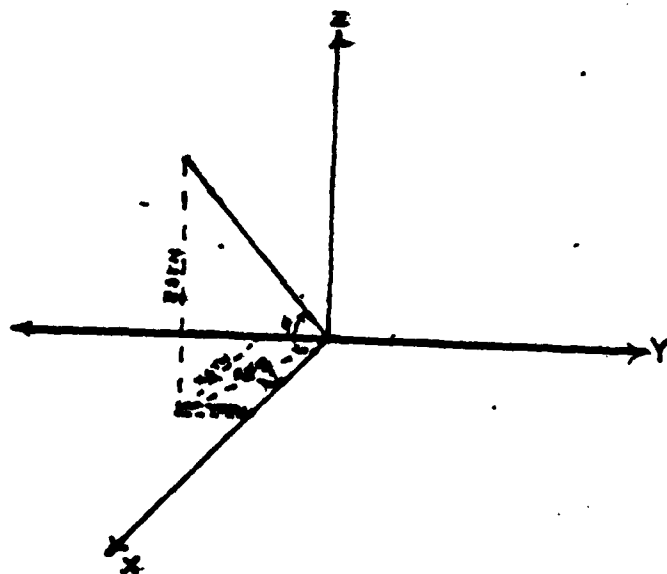


Figure 2.2-1 Geocentric Station Coordinate System

therefore,

$$(4) \quad S = \text{ARCSIN}((R_s/R_g) * \sin(90^\circ + E_c)) \quad (2.2-8)$$

$$U = 180^\circ - (90^\circ + E_c + S) \quad (2.2-9)$$

$$N = R_g * \sin U \quad (2.2-10)$$

$$M = R_g * \cos U \quad (2.2-11)$$

where M is the distance along the station position vector and N is the distance perpendicular to the station position vector that locates a point on the visibility circle.

An assumption is made that R_s is the mean semi major axis for any GPS satellite and is equal to approximately 26500 km. Another assumption is that the elevation angle cutoff for each satellite is 15 degrees.

The visibility circle is then determined by

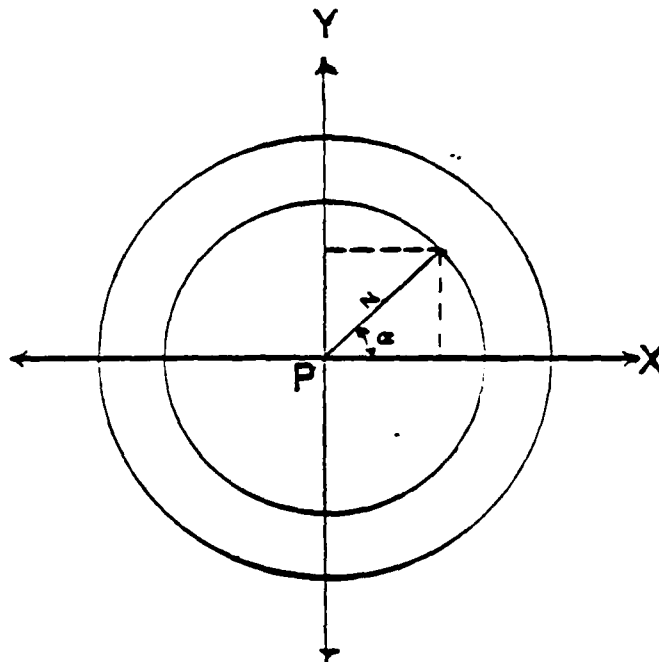


Figure 2.2-3 Overhead View of the Visibility Circle

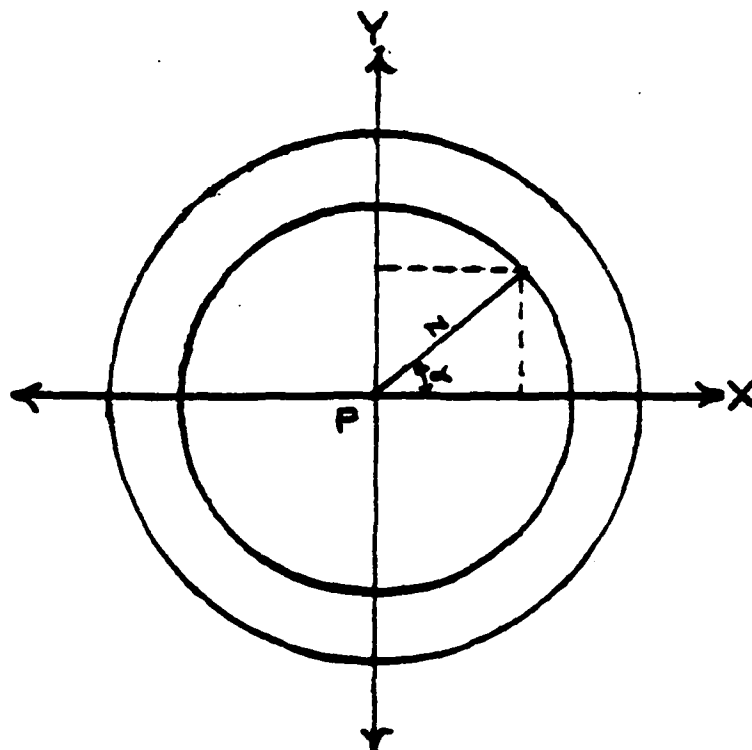


Figure 2.2-3 Polar View of the Visibility Circle

First, by solving for x , and y , in the following way:

$$x = R * \cos \alpha$$

$$y = R * \sin \alpha$$

(See Figure 2.2-3)

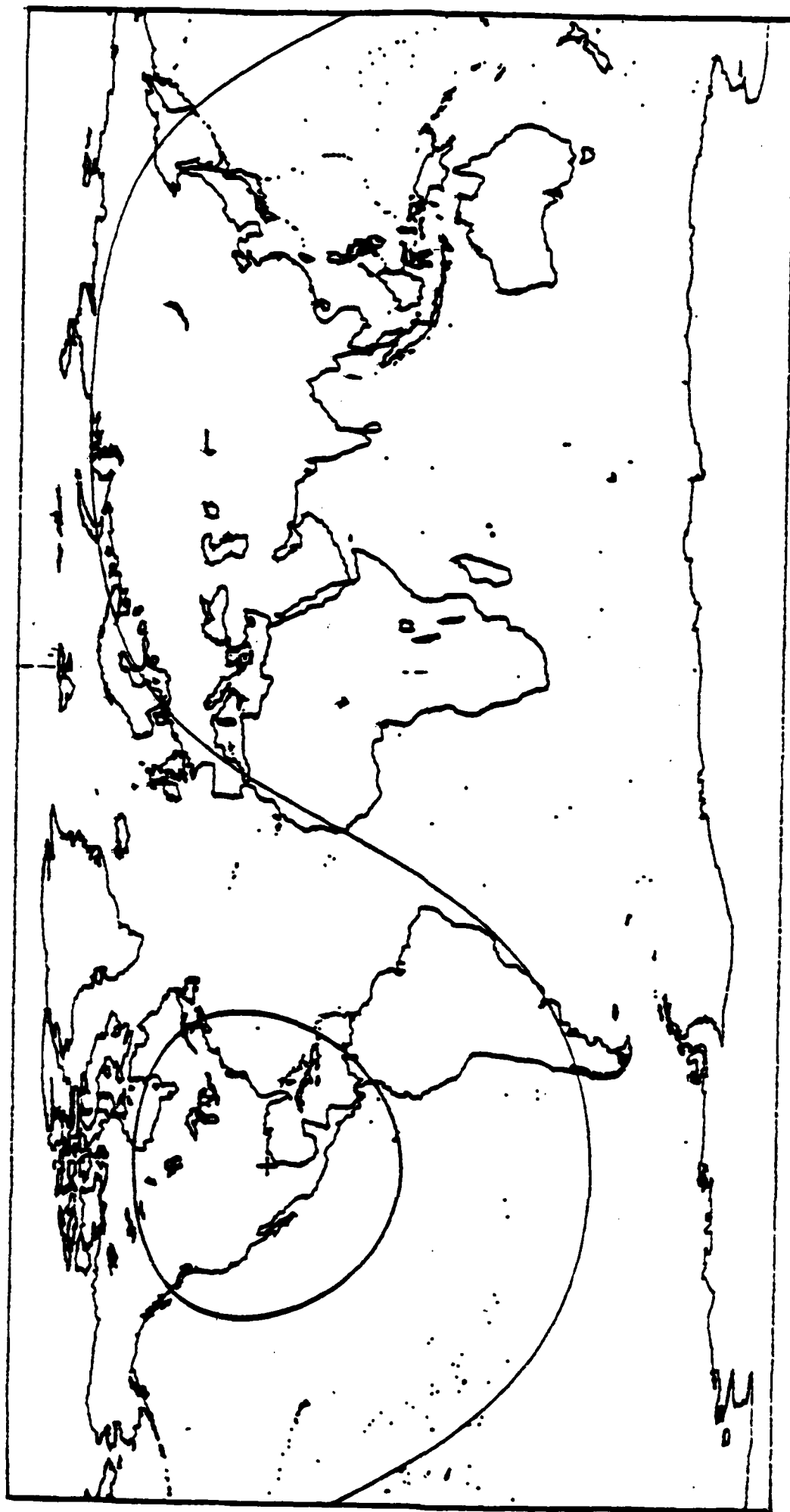
$$r = R = R_0 * \cos U$$

where α is from 0° to 360° in 1° steps.

After computing the necessary quantities, one plots the X and Y coordinates for α , (see Figure 2.3) from 0° to 360° in 1° degree steps creating the visibility circle. It is easily seen from this example that one can rotate the north pole situation by using ϕ and λ , with $R1$ and $R3$ rotation matrices. [Mueller, 1969] for plotting visibility circles for any station in the world.

Figure 2.2-4

Transit satellite visibility circle vs. GPS satellite visibility circle



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Thus, from the visibility circle plots, one has an idea of the tremendous advantage of ground station tracking GPS has over the present transit navigation system. The observer has increased time of observation and as many as 11 satellites in the 1990's depending on user location and orbit inclination to observe. [Milliken and Zoller, 1980]. (See Figure 2.2-4)

2.3 Strength of Geometry

The geometry, which is formed by the observations between the tracking station and each visible GPS satellite determines how the measurement errors are propagated to the estimated parameters in the Least Squares adjustment. The strength of the geometry to minimize the magnitude of the error propagation is measured by the Geodetic Dilution of Precision [Milliken and Zoller 1980]. This section describes the technique for a GDOP & Position Dilution of Precision (PDOP) solution.

The geometry of the GPS constellations, usually four selected satellites, contributes to the magnitude of the user accuracy and also to the potential for position errors in the GPS navigation fix. Any GPS user has to have a special interest in this fact because the four "best" satellites selected by the user receivers are those with the lowest GDOP. [Milliken and Zoller 1980].

The covariance matrix of the error, X_u , in the estimate of X_u is given by:(1)

$$\Sigma X_u = (G_u^T G_u)^{-1} G_u^T \text{Cov } \delta(A_u \bar{S} - \rho) [(G_u^T G_u)^{-1} G_u^T]^T \quad (2.3-1)$$

GDOP is calculated by setting the covariance matrix

Cov ($A_u - p$) equal to the identity matrix.

Thus, the remaining portions of (1) can then be reduced to the following:

$$\Sigma X_u = (G_u^T G_u)^{-1} \quad (2.3.-2)$$

Cov (A_u S-P) primarily reflects range measurement error statistics (i.e. satellite ephemeris, ionospheric model, and instrumentation errors), whereas, G_u , reflects only the geometry of the user systems and is the $n \times 4$ matrix of partials where n is the number of visible satellites. [Milliken and Zoller 1980].

The process for determining this purely geometric effect on a GPS user position in each axis is described below step by step.

(1) Change the station's geodetic coordinates ϕ, λ, h , to earth-fixed, earth-centered cartesian coordinates, X, Y, Z .

$$X = (N + h) \cos \phi \cos \lambda$$

$$Y = (N + h) \cos \phi \sin \lambda$$

$$Z = ((1-e^2)N + h) \sin \phi, \text{ where:}$$

N is spheroidal normal length terminated by earth's minor axis.

$$N = a / (1 - e^2 \sin^2 \phi)^{1/2},$$

and a is the semi-major axis and e the first eccentricity of the reference ellipsoid.

(2) Calculate the direction normal to ellipsoid

N is the unit vector in the direction normal to ellipsoid

$$N = \begin{bmatrix} \cos \phi \cos \lambda \\ \cos \phi \sin \lambda \\ \sin \phi \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}$$

(3) Calculate the topocentric range vector

given: X_{sta} , Y_{sta} , Z_{sta} are the earth-centered, earth-fixed cartesian coordinates of the station.

X_{sv} , Y_{sv} , Z_{sv} are the earth-centered, earth-fixed cartesian coordinates of the satellite.

then: \vec{R} is the range vector

$$\vec{R} = \begin{bmatrix} X_{sv} - X_{sta} \\ Y_{sv} - Y_{sta} \\ Z_{sv} - Z_{sta} \end{bmatrix}$$

(4) Calculate the range

given: \vec{R} is the range vector

then: $|R|$ is the range (distance from station to satellite)

$$|R| = (R_1^2 + R_2^2 + R_3^2)^{1/2}$$

$$|R| = ((X_{sv} - X_{sta})^2 + (Y_{sv} - Y_{sta})^2 + (Z_{sv} - Z_{sta})^2)^{1/2}$$

(5) Find the direction cosine (unit vector) of the range vector

given: \vec{R} is the range vector

$|R|$ is the range

then: \vec{U} is the direction cosine (unit vector) of the range vector.

$$\vec{U} = \frac{\vec{R}}{|R|} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

(6) Find the azimuth of the satellite

given: R the rotation matrix to convert XYZ to north-east-up coordinates.

$$R = \begin{bmatrix} -\cos\lambda \sin\phi & -\sin\lambda \sin\phi & \cos\phi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\lambda \cos\phi & \sin\lambda \cos\phi & \sin\phi \end{bmatrix}$$

\vec{U} is the direction cosine of the normal to the reference ellipsoid.

find:

$$(R) (\vec{U}) = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

then: az is the azimuth of the satellite

$$az = \arctan (T_2/T_1)$$

(7) Calculate the elevation angle of the satellite

given: \vec{U} and \vec{N}

$$\text{find: } \vec{U} \cdot \vec{N} = U_1 N_1 + U_2 N_2 + U_3 N_3$$

$$\text{and } \vec{U} \cdot \vec{N} = |\vec{U}| |\vec{N}| \cos \theta = \cos \theta$$

then: EA is the elevation angle of the satellite

$$EA = 90^\circ - \theta; \quad \theta = 90^\circ - EA.$$

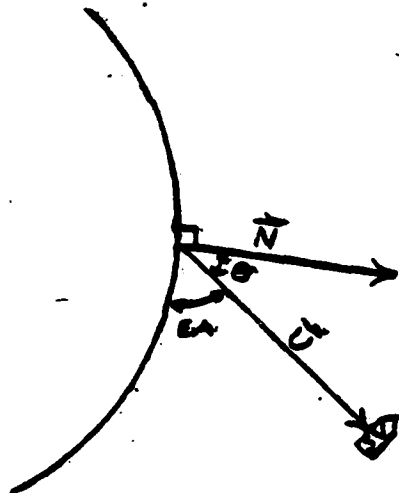


Figure 2.3-1 Geometry of Elevation Angle

$$\cos(90^\circ - EA) = U_1N_1 + U_2N_2 + U_3N_3$$

$$\sin(EA) = U_1N_1 + U_2N_2 + U_3N_3$$

Therefore, $EA = \text{ARCSIN}(U_1N_1 + U_2N_2 + U_3N_3)$

(9) Solve for Gu matrix

given: U_n is the direction cosine of the range vector of satellite n .

$b = c\Delta t$ is the range equivalent of user clock offset (Δt)

and n is the number of visible satellites.

. C is equal to the speed of light.

then: G_u is the $n \times 4$ matrix of partials

$${}^n G_{u4} = \sigma_0 \begin{bmatrix} \frac{\partial R_1}{\partial x} & \frac{\partial R_1}{\partial y} & \frac{\partial R_1}{\partial z} & \frac{\partial R_1}{\partial b} \\ \frac{\partial R_2}{\partial x} & \frac{\partial R_2}{\partial y} & \frac{\partial R_2}{\partial z} & \frac{\partial R_2}{\partial b} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial R_n}{\partial x} & \frac{\partial R_n}{\partial y} & \frac{\partial R_n}{\partial z} & \frac{\partial R_n}{\partial b} \end{bmatrix} \quad (2.3 -3)$$

where $R_n = [(x_n - x_{stn})^2 + (y_n - y_{stn})^2 + (z_n^2 - z_{stn})^2]^{1/2} + b$

and therefore $\frac{\partial R_n}{\partial b} = 1$. The σ_0 is the observation error equal to 1 meter.

where:

$$U_n = \begin{bmatrix} \frac{\partial R_n}{\partial x_n} \\ \frac{\partial R_n}{\partial y_n} \\ \frac{\partial R_n}{\partial z_n} \end{bmatrix}$$

(9) Compute Covariance matrix, $\text{COV} = \sigma_o^2 (G_u^T G_u)^{-1}$

$$\text{Cov} = \sigma_o^2 (G_u^T G_u)^{-1} = \begin{bmatrix} c_{xx}^2 & c_{xy}^2 & c_{xz}^2 & c_{xb}^2 \\ c_{yx}^2 & c_{yy}^2 & c_{yz}^2 & c_{yb}^2 \\ c_{zx}^2 & c_{zy}^2 & c_{zz}^2 & c_{zb}^2 \\ c_{bx}^2 & c_{by}^2 & c_{bz}^2 & c_{bb}^2 \end{bmatrix}$$

where c_{ij} is equal to $\rho_{ii} \sigma_i \sigma_j$ and $c_{ii} = \sigma_{ii}^2$,
 $\rho_{ii} = 1$, σ_o^2 variance of the range observation.

(10) Calculate PDOP from

$(G_u^T G_u)^{-1}$ the 4 X 4 matrix

$$\text{PDOP} = (\text{xx}^2 + \text{yy}^2 + \text{zz})^{1/2}$$

(11) Find GDOP

given $(G_u^T G_u)^{-1}$ is the 4 X 4 matrix

$$\text{then: GDOP} = (\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + \sigma_{bb}^2)^{1/2} \text{ meters}$$

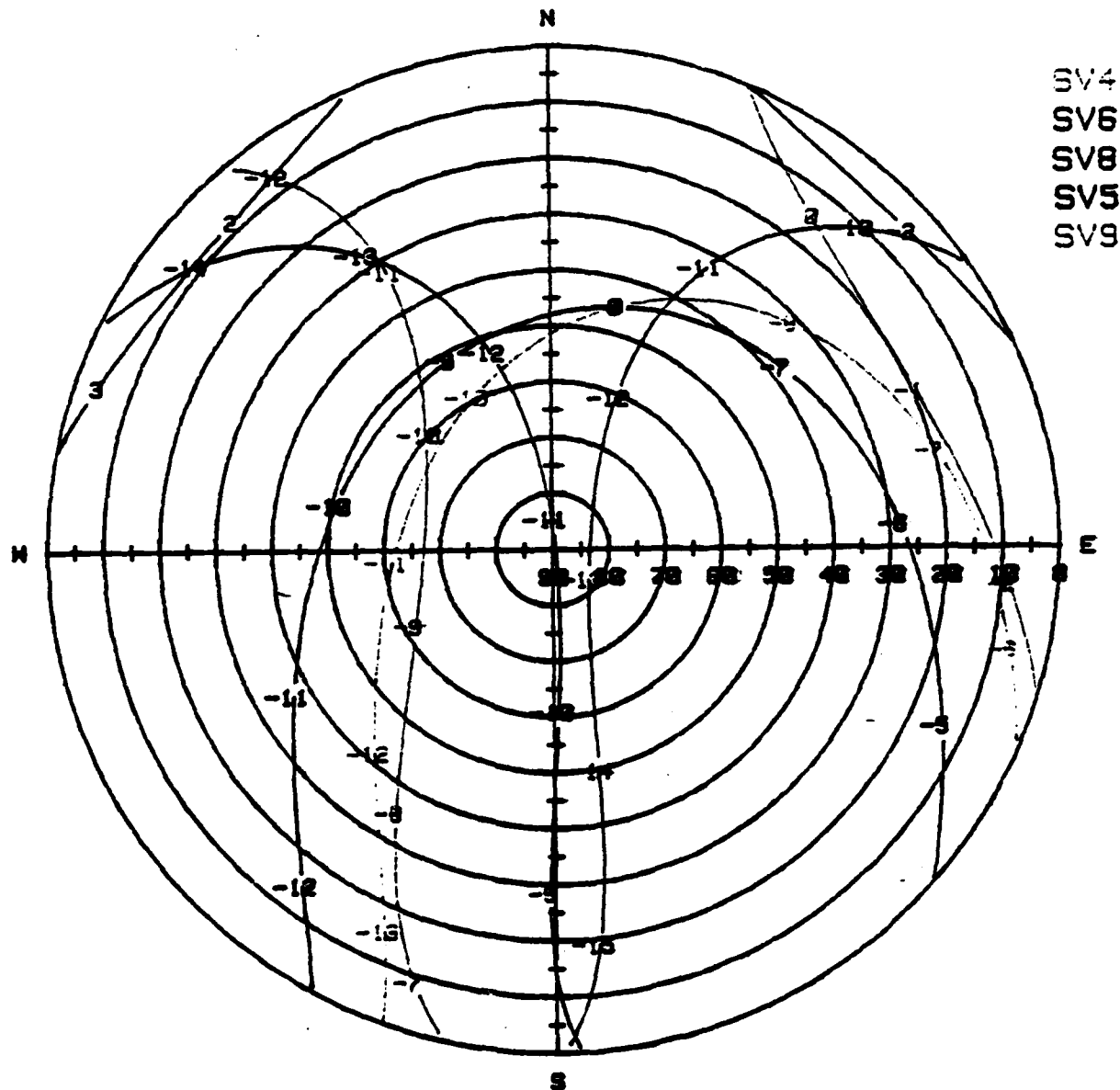
2.4 Station Tracking Geometry

The first important graph (figure 2.4-2) of Elevation angle versus time of each GPS vehicle used in this study enables the user to know the best time to track these satellites from a ground station anywhere in the world. This also verifies the GDOP values versus time by empirical observation.

These Azimuth versus Elevation Angle plots verify the GDOP and PDOP calculations by showing the two viewing periods during a 24 hour period in a "bulls eye" plot pattern (see Figure 2.4-1). This, therefore, enables the user to select the optimal

FIGURE 2.4-1

AUSTIN
AZIMUTH vs ELEVATION ANGLE



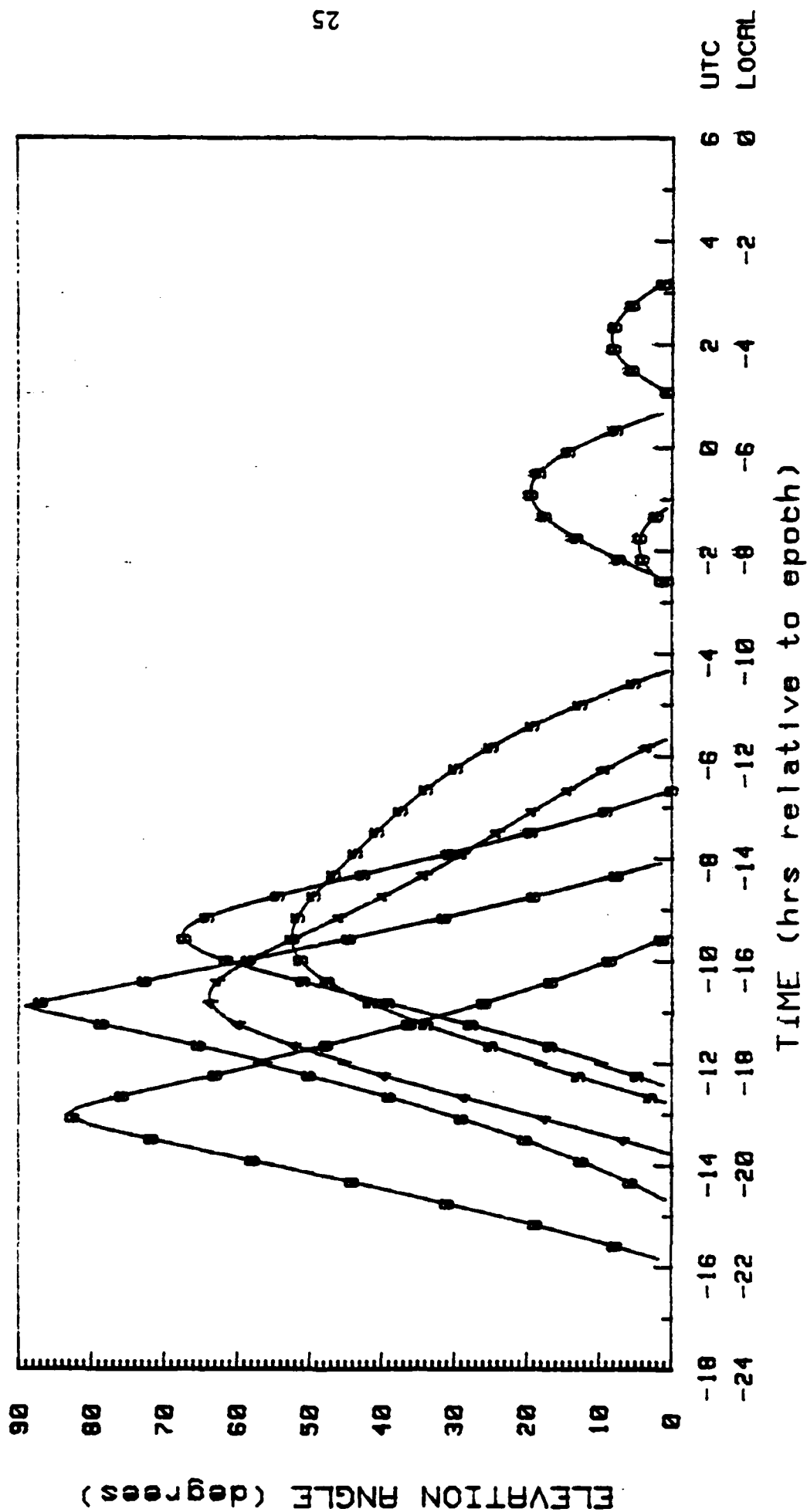
EPOCH = YEAR 83, DAY 17, SEC 0

NUMBERS ON CURVE INDICATE UTC TIME
IN HOURS RELATIVE TO EPOCH

FIGURE 2.4-2

AUSTIN

ELEVATION ANGLE VS TIME

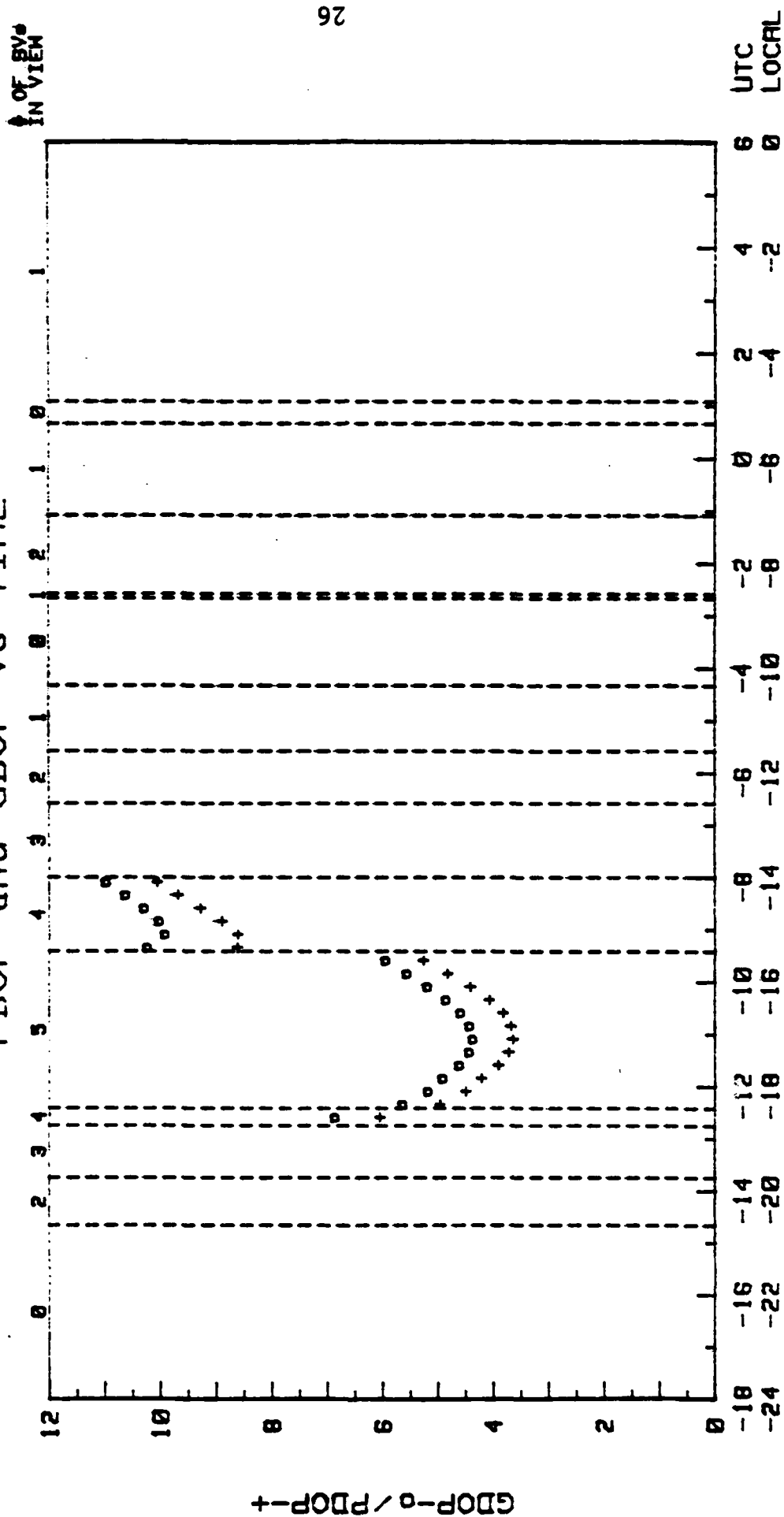


UT EPOCH = YEAR 83, DAY 17, SEC 0

FIGURE 2.4-3

AUSTIN

PDOP and GDOP vs TIME



TIME (hrs relative to epoch)

UT EPOCH = YEAR 83, DAY 17, SEC 0

3. VARIANCE-COVARIANCE STUDY

3.1 Introduction

In this chapter, the development of the least squares estimation theory and data acquisition for batch processing is described. In Section 3.2., the definitions and justifications for both the mathematical models and parameter set are presented. The technique of estimation for least squares adjustment and the procedure for linearizing the mathematical model are described in Section 3.3. Section 3.4 illustrates the procedure that utilizes the consider covariance, that is, the method for analyzing the effects of unestimated parameters. Finally, Section 3.5 describes the apriori covariance statistics and the partial derivatives used in the process to establish the covariance analysis.

The adjustment philosophy, [Bierman, 1977] was developed at the Jet Propulsion Laboratory (JPL) and was incorporated in their tracking and orbit determination section. The choice of the parameters set in our study was influenced greatly by past studies. [e.g. Fell, 1980]. Therefore, the emphasis in our study is on the selection of unestimated parameters, i.e. consider parameters and their influence on the estimate parameters. The computation of the consider covariance enables the user to estimate the errors caused by incorrect apriori statistics or mismodeling. The intention is to optimally analyze the effects of the unestimated or consider parameters so that the propagation of error covariances in the estimate parameters are readily available and can be easily computed. An important feature of

estimation algorithms is that error covariances for the estimates are readily available or can be readily computed. This covariance knowledge is used to measure confidence in the estimate parameters and discrepancies between them. The consider covariance arises in the evaluation of model that omits certain parameters, and we speak of considering the effects of these unestimated parameters, thus called consider parameters. [Bierman, 1977]

3.2 Development of The Mathematical Model

This section describes the mathematical model for the least square adjustment using estimation theory. The mathematical model is suited for GPS pseudo-range measurements and the optimal utilization of estimate-consider covariance analyses. The mathematical model is developed from previous studies [Fell, 1980], and is implemented so that it may be easily expanded to meet requirements for future use. The choice of a parameter set is justified by conclusions from previous studies about GPS pseudo-range measurements.

3.2.1 Mathematical Model

A GPS observation for geodetic point positioning requires pseudo range measurements from four GPS Space Vehicles with time being the fourth solution variable.

Let range R_i be,

$$R_i = ((u_{si} - u_{sta})^2 + (v_{si} - v_{sta})^2 + (w_{si} - w_{sta})^2)^{1/2} \quad (3.2 - 1)$$

the topocentric range from any ground station to any satellite position is shown in Figure 3.2.1, where the earth fixed coordinate system (u, v, w) is oriented towards Greenwich mean astronomical meridian (u axis) and the Conventional International Origin (w axis) with v -axis forming a right-handed coordinate system with u and w , this coordinate system being defined by Bureau International de l'Heure (BIH) [Fell, 1980, page 30].

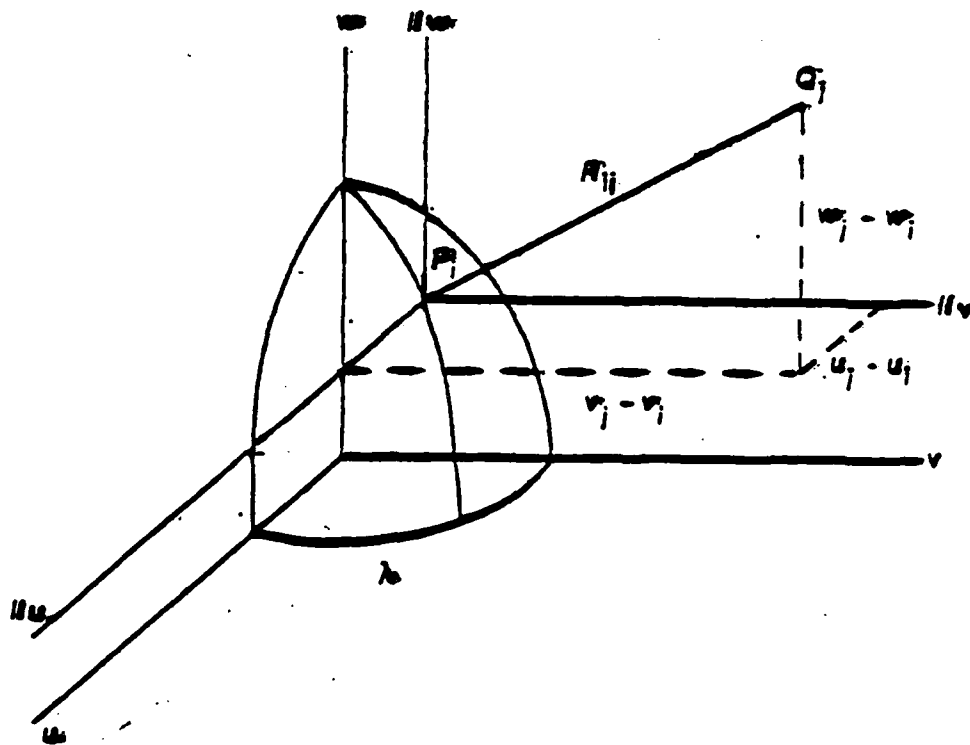


Figure 3.2-1 Geometry of Topocentric Range

The accuracy of satellite ephemerides and tropospheric refraction modeling and the stability of satellite and receiver clocks will have important consequences in the application of range observations to geodetic positioning. [Fell, 1980, Pg. 22]. From this type of statement the mathematical model adopted for this covariance analysis is formulated to result in the following equation, O_{pr} , the theoretical pseudo range observation,

$$O_{pr} = \rho + a * trop + c (b + dr * t + ag * t^2) \quad (3.2 -2)$$

where ρ is the computed topocentric range

a is scaling factor of tropospheric correction

$trop$ is the actual tropospheric correction

c = value for speed of light

$\Delta t = t - t_0$ where t_0 is the time of first observation and in this study $t_0 = 0$.

b = combined satellite-station clock bias

dr = combined satellite-station clock drift, first derivative of clock error.

ag = combined satellite-station clock aging, second derivative of clock error.

3.3 Least Squares Estimation Theory.

The utilization of the estimation theory for least squares adjustment begins with a review of the "classical" linear least squares problem. [Uotila, 1967]. Next, the linearization of a non-linear mathematical model is described.

Also, a detailed description of the partial derivatives for the Jacobian matrix or the "coefficient matrix of observation partials" [Bierman, 1977] is included.

Suppose we have a linear system,

$$z = Ax + V \quad (3.3 -1)$$

where z is an m vector of observation, x is the n vector of variables that are to be estimated, $A (m,n)$ is the 'coefficient matrix of observation partials and V is an m vector of observation errors.

The least squares solution, x_{ls} , to the linear system is chosen to minimize the mean square observation error:

$$J = \sum_{j=1}^m v_j^2 = v^T v \quad (3.3 -2)$$

$$J(x) = (z - Ax)^T (z - Ax) \quad (3.3 -3)$$

i.e.,

When more than one x minimizes the mean square error J , the x_{ls} is chosen to minimize x of smallest Euclidean length denoted as,

$$\|x\| = \left(\sum_{j=1}^n x_j^2 \right)^{1/2} \quad (3.3 -4)$$

and this uniquely defines x_{ls} .

Therefore, $J(x)$ is nonnegative and quadratic in the components of x so that a necessary condition for a minimum is that the first variation, $\delta J(x)$, vanishes.

The first variation is defined as,

$$J(x + \delta x) - J(x) = \delta J(x) + o(\delta x) \quad (3.3 -5)$$

where

$o(\delta x)$ is such that

$$\lim_{\delta x \rightarrow 0} (o(\delta x) / \|\delta x\|) = 0 \quad (3.3 -6)$$

$\delta J(x)$ is obtained by applying rules of differentiation and matrix manipulation, thus,

$$\begin{aligned} \delta J(x) &= \delta[(z - A_x)^T](z - A_x) + (z - A_x)^T \delta[z - A_x] \\ &= (-A\delta x)^T(z - A_x) + (z - A_x)^T(-A\delta x) \\ &= \delta x^T(A^T A_x - A^T z) + [\delta x^T(A^T A_x - A^T z)]^T \end{aligned} \quad (3.3 -7)$$

For the first variation $\delta J(x)$ to vanish for all variations δx , it is necessary that x satisfy the normal equations,

$$A^T A_x = A^T z \quad (3.3 -8)$$

It is important to note when $m \geq n$ and A has rank n , then $A^T A$ is non-singular and the solution, X_{ls} , is unique, i.e.,

$$X_{ls} = (A^T A)^{-1} A^T z \quad (3.3 -9)$$

Now, suppose we are given a non-linear system in the form

$$Y = f(\bar{x}_e, \bar{x}_c) + v \quad (3.3-10)$$

with nominal values assigned to the estimate and consider X_e , X_c parameters, where Y is an m vector of observations, and X is the n vector of variables that are to be estimated or considered.

The linearization of the mathematical model is obtained in a Taylor series expansion such that:

$$f(\bar{x}_e, \bar{x}_c) = f^0(\bar{x}_e^0, \bar{x}_c^0) + \frac{\partial f}{\partial (\bar{x}_e^0, \bar{x}_c^0)} \begin{bmatrix} \bar{x}_e^0 - \bar{x}_e^0 \\ \bar{x}_c^0 - \bar{x}_c^0 \end{bmatrix} + \text{H.O.T.} \quad (3.3-11)$$

(H.O.T. means higher order terms)

Writing,

$$x = \begin{bmatrix} \bar{x}_e \\ \bar{x}_c \end{bmatrix} \quad \text{and} \quad \Delta x = \begin{bmatrix} \bar{x}_e^0 - \bar{x}_e \\ \bar{x}_c^0 - \bar{x}_c \end{bmatrix}$$

we have,

$$y = f^*(x_e^0, x_c^0) + \frac{\partial f}{\partial x} \Delta x + \text{H.O.T.} + v \quad (3.3 -12)$$

Considering that x^0 is close to x the higher order terms may be neglected, thus,

$$\Delta \vec{y} = \vec{y} - f^* = \frac{\partial f}{\partial \vec{x}} \Delta \vec{x} + \vec{v} \quad (3.3 -13)$$

Now, using the notation of y for $\Delta \vec{y}$, and x for $\Delta \vec{x}$, i.e.,

$$\vec{y} = \vec{y} - f^*$$

and
$$\begin{bmatrix} A_e & A_c \end{bmatrix} = \frac{\partial f}{\partial x}$$

$$\begin{bmatrix} x_e \\ x_c \end{bmatrix} = \frac{\vec{x}_e^0 - \vec{x}_e}{\vec{x}_c^0 - \vec{x}_c}$$

We may write:

$$\vec{y} = \begin{bmatrix} A_e & A_c \end{bmatrix} \begin{bmatrix} \frac{\vec{x}_e^0 - \vec{x}_e}{\vec{x}_c^0 - \vec{x}_c} \end{bmatrix} + \vec{v} \quad (3.3 -14)$$

or

$$\vec{y} = A_e \vec{x}_e + A_c \vec{x}_c + \vec{v} \quad (3.3-15)$$

the outcome of a non-linear model for estimation theory of least squares adjustment.

3.4 Development of Consider Covariance.

In this section is presented the derivation of the consider covariance that arises when we speak of considering the effects of the unestimated parameters, consider parameters, x_c . The

estimate parameters, X_e , and the truth model being defined as

$$\bar{y} = A_e \bar{x}_e + A_c \bar{x}_c + \bar{v} \quad (3.3-16)$$

with the assumption that A_e , A_c , X_e are deterministic. Therefore,

$$E(x_e) = X_e = \text{a constant.}$$

Also, assume that X_c and v are random variables with known covariance and the equations have been normalized to show the following characteristics so that

$$E(v) = 0 \text{ and } E(vv^T) = \Sigma_v \quad (3.3-17)$$

where Σ_v is the m dimensional identity covariance on measurements.

Let $E(\bar{x}_c) = \bar{\mu}_c$, where $\bar{\mu}_c$ is the average value of the variable over all possible values.

$$\text{Then, } E[(\bar{x}_c - \bar{\mu}_c)(\bar{x}_c - \bar{\mu}_c)^T] = \Sigma_c \quad (3.3-18)$$

where Σ_c is the consider covariance.

By simple manipulation of 3.3-18 we get:

$$E(\bar{x}_c \bar{x}_c^T) = \Sigma_c + \bar{\mu}_c \bar{\mu}_c^T \quad (3.3-19)$$

We assume that $E(v \bar{x}_c^T) = 0$, that is, there is no correlation between the errors on observations and the consider parameters.

Let the filter model be,

$$\bar{y} = A_e \bar{x}_e + \bar{v} \quad (3.3-20)$$

and from the derivation of the least squares solution we have,

$$\bar{x}_e = [A_e^T \Sigma_v^{-1} A_e]^{-1} A_e^T \Sigma_v^{-1} \bar{y} \quad (3.3-21)$$

$$\begin{aligned} &= [(A_e^T \Sigma_v^{-1} A_e)^{-1} A_e^T \Sigma_v^{-1}] (A_e \bar{x}_e + \bar{v}) \\ &= [(A_e^T \Sigma_v^{-1} A_e)^{-1} A_e^T \Sigma_v^{-1}] (A_e \bar{x}_e + A_c \bar{x}_c + \bar{v}) \\ &= [A_e^T \Sigma_v^{-1} A_e]^{-1} A_e^T \Sigma_v^{-1} A_e \bar{x}_e + [A_e^T \Sigma_v^{-1} A_e]^{-1} A_e^T \Sigma_v^{-1} A_c \bar{x}_c + \dots \\ &\quad \dots + [A_e^T \Sigma_v^{-1} A_e]^{-1} A_e^T \Sigma_v^{-1} \bar{v} \end{aligned} \quad (3.3-22)$$

If we now define the following matrices,

$$\hat{P}_e = [A_e^T \Sigma_v^{-1} A_e]^{-1} \quad (3.3-23)$$

$$\hat{S}_c = P_e A_e^T \Sigma_v^{-1} A_c \quad (3.3-24)$$

$$\hat{S}_v = P_e A_e^T \Sigma_v^{-1} \quad (3.3-25)$$

then (3.3.-17) reduces to:

$$\begin{aligned} \hat{x}_e &= \hat{P}_e \hat{P}_e^{-1} \bar{x}_e + \hat{S}_c \bar{x}_c + \hat{S}_v \bar{v} \\ \hat{x}_e - \bar{x}_e &= \hat{S}_c \bar{x}_c + \hat{S}_v \bar{v} \end{aligned} \quad (3.3-26)$$

Examining the statistical characteristics of random variable X ; we have

$$E[\hat{x}_e - \bar{x}_e] = E(\hat{x}_e) - \bar{x}_e = \hat{S}_c E(\bar{x}_c) + \hat{S}_v E(\bar{v})$$

from the fact that $E(c) = c$ where c is a constant.

If X_c is biased, i.e., $E(X_c) \neq 0$, then X_e is a biased estimate,

$$\text{therefore, } E(\hat{x}_e) - \bar{x}_e = \hat{S}_c \hat{u}_c .$$

Since the estimate covariance may be defined as,

$$\begin{aligned}
 \Sigma \hat{x}_e &= E[(\hat{x}_e - x_e)(\hat{x}_e - x_e)^T] && \text{using equation (3.3-26)} \\
 &= E[(\hat{S}_c \hat{x}_c + \hat{S}_v \hat{v})(\hat{S}_c \hat{x}_c + \hat{S}_v \hat{v})^T] \\
 &= E[\hat{S}_c \hat{x}_c \hat{x}_c^T \hat{S}_c^T + \hat{S}_c \hat{x}_c \hat{v}^T \hat{S}_v^T + \hat{S}_v \hat{v} \hat{x}_c^T \hat{S}_c^T + \hat{S}_v \hat{v} \hat{v}^T \hat{S}_v^T] \\
 &= \hat{S}_c E(\hat{x}_c \hat{x}_c^T) \hat{S}_c^T + \hat{S}_c E(\hat{x}_c \hat{v}^T) \hat{S}_v^T + \hat{S}_v E(\hat{v} \hat{x}_c^T) \hat{S}_c^T + \hat{S}_v E(\hat{v} \hat{v}^T) \hat{S}_v^T \\
 &= \hat{S}_c (\Sigma_c + \bar{u}_c \bar{u}_c^T) \hat{S}_c^T + \hat{S}_v \Sigma_v \hat{S}_v^T && \text{using equations (3.3-19)} \\
 &= \hat{S}_c \Sigma_c \hat{S}_c^T + \hat{S}_c \bar{u}_c \bar{u}_c^T \hat{S}_c^T + \hat{P}_e A_e^T \Sigma_v^{-1} \Sigma_v \Sigma_v^{-1} A_e \hat{P}_e && \text{and (3.3-15)} \\
 &= \hat{S}_c \Sigma_c \hat{S}_c^T + \hat{S}_c \bar{u}_c \bar{u}_c^T \hat{S}_c^T + \hat{P}_e \hat{P}_e^{-1} \hat{P}_e \\
 &&& \text{using equation (3.3-25)}
 \end{aligned}$$

therefore, the estimate covariance may be written in the form,

$$\Sigma \hat{x}_e = \hat{S}_c \Sigma_c \hat{S}_c^T + \hat{S}_c \bar{u}_c \bar{u}_c^T \hat{S}_c^T + \hat{P}_e \quad (3.3-24)$$

and Σ_c defined in equation (3.3-24)

If x_c is unbiased, i.e., $\bar{u}_c = \phi$

Therefore, the total covariance is finally derived to include the consider covariance,

$$\Sigma \hat{x}_e = \hat{P}_e + \hat{S}_c \Sigma_c \hat{S}_c^T \quad (3.3-25)$$

3.5 Partial Derivatives

Now that the covariances have been derived, we need to consider the partial derivatives for the Jacobian matrix. The interactive program is capable of handling multiple variables from the mathematical model for pseudo range of measurements. This section describes the technique for solving the adjustment algorithm involved in covariance analysis. In the following.

sections, the discussion will contain information about certain weight matrices, apriori statistics, and partial derivatives involved in the method for the estimation theory.

Assume O_{pr} , theoretical pseudo range measurement, see equation (3.2.-1) to be given by the mathematical model,

$$O_{pr} = \rho + \alpha * trop + c (b + dr * t + ag * t^2) \quad (3.5-1)$$

where the variables are defined in Section 3.2.

The list of parameters for this particular study is as follows:

NSTA latitude of station (meters)

ESTA longitude of station (m)

USTA height of station (m)

TROP tropospheric correction (m)

LSAT long-track of satellite (tangential) (m)

CSAT cross-track of satellite (m)

RSAT radial-track of satellite (m)

BIAS combined user and satellite clock bias (Sec)

DRIFT combined user and satellite clock drift (Sec/sec)

AGEING combined user and satellite clock aging (Sec/sec²)

This study uses the NSTA, ESTA, USTA as estimate parameters and shows the correlation between any of the other parameters and their effect on the NSTA, ESTA, and USTA for geodetic positioning techniques specifically pseudo range. The impact of consider parameters onto the estimable parameters and the numerical degradation (failure) in estimate correlated parameters.

3.5.1 Partial Derivatives Development.

We first consider the measurement model involving pseudo-range, PR,

$$PR = O_{pr} = \rho + c \cdot \text{trop} + c (b + dr \cdot t + ag \cdot t^2)$$

The partials are computed in a subroutine PDERIV and stored in an array for each iteration in the following fashion. (Chain rule)

$$\begin{bmatrix} \frac{\partial O_{pr}}{\partial NSTA} \\ \frac{\partial O_{pr}}{\partial ESTA} \\ \frac{\partial O_{pr}}{\partial USTA} \end{bmatrix} = \underset{1}{\begin{bmatrix} \frac{\partial O_{pr}}{\partial XSTA} \end{bmatrix}} \underset{3}{\cdot} \underset{3}{\begin{bmatrix} \frac{\partial XSTA}{\partial NSTA, ESTA, USTA} \end{bmatrix}} \quad (3.5-2)$$

where XSTA is the position vector of the station, these partials result in being equal to

$$= \underset{1}{\begin{bmatrix} -L.O.S. \end{bmatrix}} \underset{3}{\overset{T}{\cdot}} \underset{3}{\begin{bmatrix} ROTSTA \end{bmatrix}} \quad (3.5-3)$$

where line of sight, L.O.S., is equal to a unit vector, or sometimes referred to as the direction cosine, and is defined as a normalized vector,

$$L.O.S. = \frac{R_{sv} - R_{sta}}{\text{length of } (R_{sv} - R_{sta})} \quad (3.5-4)$$

where RSV and RSTA are the position vectors of satellite and station, respectively. ROTSTA is a rotation matrix transforming North, East, and Up coordinates (local station coordinate systems see Figure 3.5-1b) to earth-fixed cartesian.

The next partial of the mathematical model, , that is, the partial of Opr w.r.t alpha, α , the scale factor of the tropospheric correction is,

$$\frac{\partial \text{Opr}}{\partial \alpha} = \text{TROP} \quad (3.5-5)$$

where TROP is the computed tropospheric correction and is found using the CHAO model.(1983) The CHAO model (Reference, 1983) was tested and evaluated by the GST department at DMA.

The partials for long-track, cross-track and radial-track consider parameters are computed in a similar method as the ground station ones. Thus, we have,

$$\begin{bmatrix} \frac{\partial \text{Opr}}{\partial \text{LSAT}} \\ \frac{\partial \text{Opr}}{\partial \text{CSAT}} \\ \frac{\partial \text{Opr}}{\partial \text{RSAT}} \end{bmatrix} = {}_1 \begin{bmatrix} \frac{\partial \text{Opr}}{\partial \text{XSAT}} \end{bmatrix}_3 \cdot {}_3 \begin{bmatrix} \frac{\partial \text{XSAT}}{\partial \text{LSAT, CSAT, RSAT}} \end{bmatrix}_3 \quad (3.5-6)$$

$$\text{where SATROT} = {}_3 \begin{bmatrix} \vdots \\ 1 \\ \vdots \\ c \\ \vdots \\ r \\ \vdots \end{bmatrix}_3 \quad (3.5-7)$$

and l, c, r are the components for a local satellite coordinate system. (see figure 3.5-1a)

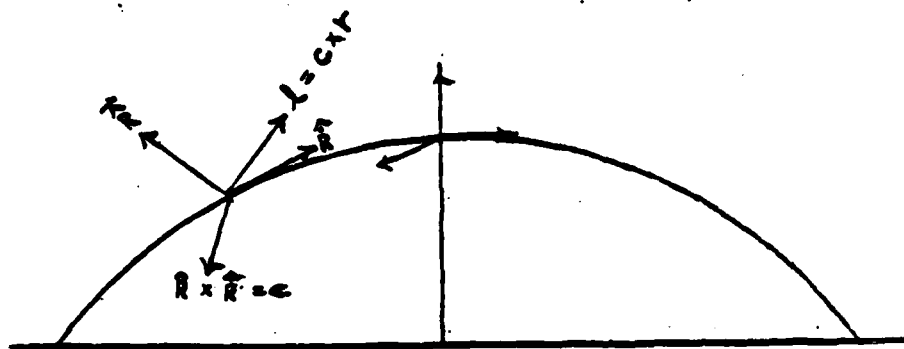


FIGURE 3.5-1a

Local satellite coordinate system

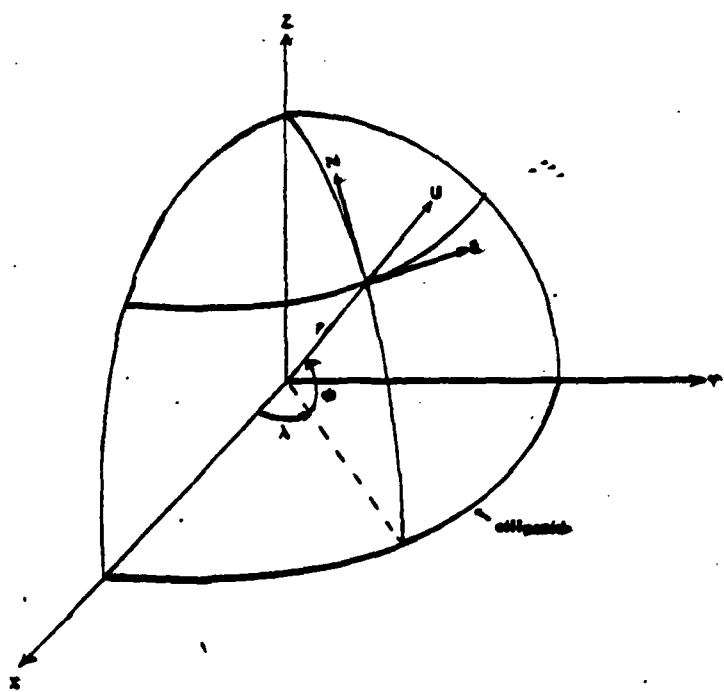


FIGURE 3.5-1b

Local station coordinate system

The process for the computation is as follows, where $\hat{R} = \frac{R}{|R|}$ that is, \hat{R} , the unit vector of the satellite, where $R = (x, y, z)$ of the satellite.

Next, set up a vector, $\hat{\dot{R}} = \frac{\dot{R}}{|\dot{R}|}$ where $\dot{R} = (\dot{x}, \dot{y}, \dot{z})$ of the satellite. that is, the unit velocity vector of the satellite.

The process continues by producing the cross product of $\hat{R} \times \hat{\dot{R}}$.

$$\text{which is } c = \hat{R} \times \hat{\dot{R}}$$

Finally,

$$l = c \times r \text{ (cross product)}$$

Thus we have,

$$\begin{bmatrix} \frac{\partial \rho_{pr}}{\partial LSAT} \\ \frac{\partial \rho_{pr}}{\partial CSAT} \\ \frac{\partial \rho_{pr}}{\partial BSAT} \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}^T \cdot \begin{bmatrix} 1 & c & r \end{bmatrix} \quad (3.5-8)$$

The partials for the bias, drift, and aging terms are easily found. Therefore,

$$\frac{\partial \rho_{pr}}{\partial BIAS} = 1, \quad \frac{\partial \rho_{pr}}{\partial DRIFT} = t, \quad \frac{\partial \rho_{pr}}{\partial AGING} = t^2$$

where t represents the time of the pseudo range measurement.

3.6.2 Apriori covariance matrices and apriori uncertainties for measurement types and parameters.

The weight matrix, called COVV, is set up for the three measurement types pseudo range, range differencing, and second range differencing is as follows:

$$i_m \text{ COVV } i_m = \begin{bmatrix} \sigma_{i_m}^2 & 0 & 0 \\ 0 & \sigma_{i_m+1}^2 & 0 \\ 0 & 0 & \sigma_{i_m+2}^2 \end{bmatrix} \quad (3.5-9)$$

where $Im = NMEAS = 1$ the pseudo range measurement type for this study. The interactive program requires the number of measurement types for the adjustment algorithm and may account for all three measurement types in the future. $COVE\emptyset$ is the apriori covariance matrix for estimate parameters and is formed in this fashion,

$$COVE\emptyset = \begin{bmatrix} \sigma_j^2 & & & 0 \\ & \ddots & & \\ 0 & & \ddots & \\ & & & \sigma_n^2 \end{bmatrix} \quad (3.5-10)$$

where $j = 1, \dots, n$ the number of estimate parameters.

$COVC$ is the apriori covariance matrix of consider parameters, and is formed this way,

$$COVC = \begin{bmatrix} \sigma_1^2 & & & 0 \\ & \ddots & & \\ 0 & & \ddots & \\ & & & \sigma_n^2 \end{bmatrix} \quad (3.5-11)$$

where $i = 1, \dots, n$ the number of consider parameters.

The a priori uncertainties for the measurement types and parameters were found by investigating recent studies (personal communication) performed by Naval Surface Weapons Center, NSWC, Dahlgren, Virginia. There is a table of these apriori values found in the appendix.

4. EXPERIMENTAL RESULTS

4.1. Test Plan Design

The experiments involved in this study show the effects of each consider parameter individually onto the estimate parameters by means of a consider covariance. The experimental test plan for this study involved the performance of fifteen experiments with results displayed on graphs. The least squares adjustment algorithm allows the user to chose estimate and consider parameters. The first seven experiments in this study use the tracking station Austin, Texas and the estimate parameter are NSTA, ESTA, and USTA which in the E-N-U coordinate system is [Conley, 1984] is adopted for this study. The experiments use the five GPS vehicles currently operational with the pseudo random noise (PRN) numbers 4,5,6,8 & 9. The epoch of the orbits is the same for all orbits Day 17, 0 seconds, 1983.

The first experiment involves the solution set of the latitude (N), longitude (E) and vertical (U) components of the local coordinate system for the station Austin, Texas. In Table 1, experiment number 1 the results show the sigmas of latitude, longitude, and vertical being .1, .2 and .1 meters respectfully.

The second experiment calls for the solution set of estimate parameters NSTA, ESTA & USTA and shows the effects of the consider parameter, TROP, onto the estimate parameters, tropospheric correction by means of the consider covariance (formula 3.3-1). This experiment shows what happens to each

estimate parameter when the tropospheric correction is considered, thus the name consider parameter. Each experiment solves for NSTA, ESTA and USTA and uses the remaining parameters in the model such as ephemerides parameters (LSAT, RSAT, CSAT) and combined satellite-station clock parameters (BIAS, DRIF, AGNG) plus the tropospheric correction individually as consider parameters. The model of pseudo-range observations from tracking satellite to station is the one developed in this study as formula 3.2-3. The observation error used for pseudo range is assumed to be 1 meter. The assumed standard deviations used in the experimental tests from the list of parameters on page 30 are as follows:

NSTA	=	50 meters
ESTA	=	50 meters
USTA	=	50 meters
LSAT	=	8 meters
RSAT	=	2 meters
CSAT	=	4 meters
BIAS	=	10^{-5} sec.
DRIF	=	10^{-10} sec/sec
AGNG	=	10^{-18} sec/sec ²

The test results from the experiments indicate the reliability of GPS positioning with respect to changes in the geometry with time and the effects of consider parameters. The changes in geometry were indirectly measured by the changes in the standard deviation of the estimate parameters and changes in the correlation coefficients among the estimate parameters.

To summarize, the following factors stipulate each experiment:

***Constants**

- five GPS satellites
- tracking station Austin, Texas
- Pseudo-range observation error, 1 meter.
- Elevation angle cutoff 15 degrees.

***Estimate parameters in the experiments ESTA, NSTA, USTA,**

***Consider or unestimated parameters are satellite long, cross and radial track, and tropospheric correction, and clock parameters bias, drift, and aging.**

4.2 Test Plan Sequence for Each Experiment.

Preprocessor activity for each experiment begins with the position, velocity, and ground trace data for each satellite. The azimuth and elevation angles versus time of the range observations from each satellite are computed and stored in files using the UNIVAC 1100 series computer. This data is used in the station geometry study and found to be useful in the evaluation and analysis of each experiment. The files are then transformed into plotable files for the Hewlett Packard HP 9845 desktop computer. The output display is found in Section 2.2 page .The PDOP and GDOP plots as a function of time show that the number of satellites in view and the tracking time for the receiver is sufficient for a solution. The GDOP study shows the user that the optimal time for satellite tracking is from 16^d 12^h to 16^d 16^h.

4.3 Processor Activity for Each Experiment.

The parameters NSTA, ESTA and USTA remain estimate parameters for each experiment and the experiments are comprised of the consider parameter set using one at a time making seven experiments. For instance, the second experiment shows the effect of tropospheric correction (TROP) onto the estimate parameters NSTA, ESTA, and USTA.

The second experiment results found in Table 1 shows a 0.1 to 0.3 meter difference in the standard deviation for the latitude component NSTA. The standard deviation of ESTA shows a .3 to .8 meter difference caused by the consider parameter, tropospheric correction. The USTA standard deviation took the a change being from .4 meter to .5 meters.

The third experiment on Table 1 is the NSTA, ESTA, and USTA as estimate parameters and long track of the satellite (LSAT) as a consider parameter. In the final solution there is 1.2 meter difference shown on the NSTA parameter. The same result is found in the ESTA parameter. The major difference found in the USTA parameter is .8 meter caused by the consider parameter, LSAT.

The fourth experiment on Table 1 is the NSTA, ESTA, USTA as estimate and cross-track of satellite (CSAT) being the consider parameter. The consider parameter, CSAT, had an affect on the final solution for NSTA from 1.6 to 2.2 meters. The consider parameter showed approximately a .5 meter noise level in the estimate parameter ESTA. The consider parameter had a negligible effect on the USTA estimate parameter.

The fifth experiment on Table 2 involving the consider parameter, radial-track of satellite, (RSAT) affected the estimate parameter by approximately .2 to .8 meter. The ESTA parameter took a 1.0 to 2.2 meter jump from the RSAT consider parameter. The RSAT has a 0.4 to 1.8 affect on the USTA parameter.

The next experiments involving the BIAS, DRIFT and AGING consider parameters revealed the most significant influence on the state parameters.

The fifth experiment involves the station-satellite BIAS clock consider parameter. The most significant errors are found in the state parameters from this consider parameter. All three estimate parameters show a degradation but the USTA parameter took the most "beef" from the consider parameter.

The sixth experiment involves the consider parameter, clock drift, DRIF, in which a 10 meter fluctuation in the state parameter NSTA. The ESTA parameter shows a 10m fluctuation due to the consider parameter. The USTA has the most significant fluctuation with a 600 meter difference.

In the seventh experiment the station-satellite clock aging shows negligible fluctuation from the state parameters. The only parameter that shows a .1 meter fluctuation due to this consider parameter is the USTA parameter and a station tracking geometry useful for simulation studies. The point is made that there is a level of confidence added to the measures of GDOP. At this level the study is very useful for setting up tracking

stations and testing receiver capabilities. The chapter is written in a fashion so that an inexperienced scientist may understand the process for computing azimuth and elevation angles of a satellite from a station on the earth's surface. Also, there is a short description on the GEOSTAR receiver currently being tested at the University of Texas by Applied Research Laboratory (ARL) at Austin, Texas.

4.4 Correlation Coefficients

Experiments nine through fifteen found on Tables 3 and 4 show the interrelationships among the parameters involved in the math model for pseudo range. A value of unity indicates that the parameters are highly dependent and therefore produces an unreliable system for improvements in station coordinates for point-positioning using the pseudo range technique.

This study found a high correlation among certain parameters such as the tropospheric correction has a .8 correlation coefficient with the vertical component (Table 3). Another significant finding is a positive correlation coefficient with cross-track of satellite and the longitude and vertical components of the station. Also in Table 3 the radial-track of satellite and the vertical component of the station have a .9 correlation coefficient. Finally, in Table 4 there is a high correlation coefficient between bias, drift clock parameters and the vertical component of the station.

5. CONCLUSIONS AND FUTURE RESEARCH

5.1 Summary and Conclusions

A GPS Pseudo-range covariance analysis of the interactive program is presented in Appendix A for station-tracking geometry, experiment planning, and future design analysis. Tables and figures for ease of operation and adaptation are presented. The program itself is well documented for future users (example; expanding parameter set). By an explanation of the theory on which the program is based and of the mathematical models which it incorporates, an overview of GPS pseudo-range process is given.

An introductory chapter touches upon the historical and future aspects of GPS as well as its application to point-positioning.

Chapter 2 opens with the process for determining the preprocessor stages that is the generation of the GPS satellite ephemeris, the subsatellite points projected on a world map, azimuth and elevation-angle plots of the satellite from ground station versus time, and the GDOP study. A description of the method for computing azimuth and elevation angle of a satellite from a ground station is presented completely in a tutorial fashion. The development of the geodetic receiver (GEOSTAR) is presented along with a broad description of its capabilities.

In Chapter 3, the development of the least squares estimation and the method used for the covariance analysis is described. The process and mathematical models are described for the development of consider covariance and the partial

derivatives. Clearly more research is needed to improve the mathematical models and the estimation process. In the next section, an attempt is made to qualify the future areas of research.

5.2 Suggestions for future research.

This research will no doubt continue since the need for accurate and timely data analysis is accelerating. several suggestions that may help to enhance the future needs are: 1) to test different elevation angle cutoffs for the best observations and 2) to experiment with the tropospheric corrections models. Additionally, there could be an attempt to determine the relationship between time bias and the solution for height of a station along with different elevation angle cutoffs. One could study the effect of 3rd order ionospheric errors and how they interact with the estimate parameters. Also, a question arises as to the advantages pseudo-range measurements have over Doppler or range differencing methods. As more satellites are launched, there should be an attempt to have a selective method to obtain the "best" satellite observations from passive tracking stations. Further consideration should be given to the cost effectiveness of these proposals for GPS point positioning.

TABLE 1

Variations in σ when GDOP is minimum

EXPERIMENT	ESTIMATE PARAMETER	CONSIDER PARAMETER	σ_N	σ_E	σ_U
1	N E U		0.1	0.2	0.1
2	N E U	T	$0.1 < \sigma < 0.3$	$0.3 < \sigma < 0.8$	$0.4 < \sigma < 0.5$
3	N E U	L	$1.2 < \sigma < 1.8$	$1.6 < \sigma < 2.9$	$1.0 < \sigma < 1.8$
4	N E U	C	$1.5 < \sigma < 2.2$	$0.4 < \sigma < 2.0$	$1.4 < \sigma < 2.0$

N Latitude Component
 E Longitude Component
 U Vertical Component
 T Tropospheric correction
 L Long-track of satellite
 C Cross-track of satellite
 R Radial-track of satellite
 (σ units in meters)

TABLE 2

Variations in σ when GDOP is minimum

EXPERIMENT	ESTIMATE PARAMETER	CONSIDER PARAMETER	σ_N	σ_E	σ_U
5	N E U	R	$0.2 < \sigma < 0.8$	$1.0 < \sigma < 2.2$	$0.4 < \sigma < 1.8$
6	N E U	B	$10 < \sigma < 310$	$10 < \sigma < 480$	$610 < \sigma < 860$
7	N E U	D	$20 < \sigma < 50$	$20 < \sigma < 100$	$690 < \sigma < 820$
8	N E U	A	$0.1 < \sigma < 0.2$	$0.1 < \sigma < 0.2$	$0.1 < \sigma < 0.2$

N Latitude Component
 E Longitude Component
 U Vertical Component
 R Radial-track of satellite
 B Clock Bias
 D Clock Drift
 A Clock aging

(σ units in meters)

TABLE 3

Variations in correlation coefficients ρ when GDOP is minimum

EXPERIMENT	ESTIMATE PARAMETER	ρ (1,2)	ρ (1,3)	ρ (1,4)	ρ (2,3)	ρ (2,4)	ρ (3,4)
9	N (1)	$-.2 < \rho < .4$	$-.2 < \rho < -.1$	$-.7 < \rho < -.3$	$-.6 < \rho < .1$	$-.5 < \rho < .4$	$.6 < \rho < .8$
	E (2)						
	U (3)						
	T (4)						
10	N (1)	$0 < \rho < .6$	$0 < \rho < .5$	$.3 < \rho < .7$	$.5 < \rho < .6$	$.5 < \rho < .7$	$.5 < \rho < .6$
	E (2)						
	U (3)						
	L (4)						
11	N (1)	$-.5 < \rho < -.3$	$.5 < \rho < .7$	$-.7 < \rho < -.5$	$-.5 < \rho < .1$	$.7 < \rho < .8$	$.7 < \rho < .8$
	E (2)						
	U (3)						
	C (4)						
12	N (1)	$-.2 < \rho < .5$	$-.4 < \rho < .1$	$-.5 < \rho < .1$	$-.8 < \rho < -.2$	$-.7 < \rho < -.5$	$.8 < \rho < .9$
	E (2)						
	U (3)						
	R (4)						

N Latitude Component
 E Longitude Component
 U Vertical Component
 T Tropospheric correction
 L Long-track of satellite
 C Cross-track of satellite
 R Radial-track of satellite

(ρ unitless)

14	N (1)	$-.2 < \rho < .6$	$-.4 < \rho < -.2$	$.2 < \rho < .4$	$-.6 < \rho < -.2$	$.6 < \rho < .2$	$-.9 < \rho < -.7$
	E (2)						
	U (3)						
	D (4)						
15	N (1)	$-.2 < \rho < .8$	$-.3 < \rho < 0$	$-.3 < \rho < -.2$	$-.2 < \rho < .2$	$-.4 < \rho < -.2$	$.1 < \rho < .6$
	E (2)						
	U (3)						
	A (4)						

N Latitude Component
 E Longitude Component
 U Vertical Component
 B Clock Bias
 D Clock Drift
 A Clock Aging

(ρ unitless)

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